LOYOLA UNIVERSITY CHICAGO

Department of Mathematics and Statistics
Lake shore campus | 1032 W. Sheridan Road | Chicago, Illinois 60660
Phone 773.508.3558 | Fax 773.508.2123 | www.math.luc.edu

Math 131 - Spring 2025 - Common Final Exam, version A

Print name:____

Section number:_______ **Instructor's name:**________________________

Directions:

- This exam has 14 questions. Please check that your exam is complete, but otherwise keep this page closed until the start of the exam is called.
- Fill in your name, and your instructor's name.
- It will be graded out of 92 points.
- Show your work. Answers (even correct ones) without the corresponding work will receive no credit.
- You may use a calculator which cannot connect to the internet. The use of any notes or electronic devices other than a calculator is prohibited.
- A formula sheet has been provided with this exam. You may not refer to any other notes during the exam.

Good luck!

Question:	1	2	3	4	5	6	7	8
Points:	6	3	6	7	12	11	10	6
Score:								

Question:	9	10	11	12	13	14		Total
Points:	5	6	4	4	6	6		92
Score:								

1. Sand is continuously falling onto a pile. The height of the pile, h (in meters), is measured at various times, t (in hours).

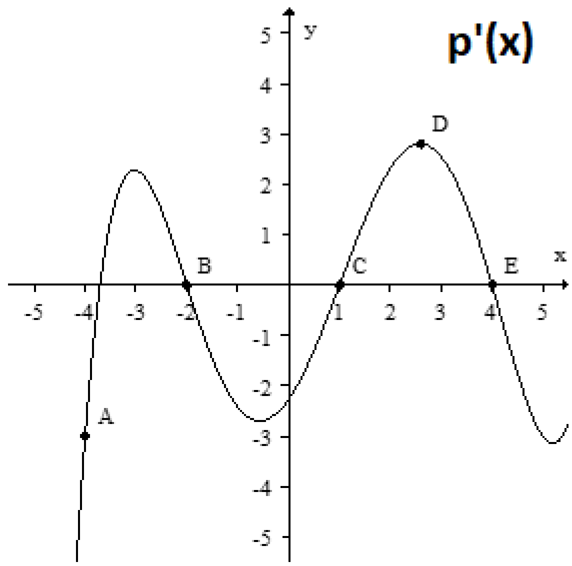
t	3.20	3.40	3.60	3.80	4.00
$h(t)$	4.73	5.63	5.91	6.12	6.22

(a) (3 points) What is the **average rate of change** in the height of the sand pile from $t = 3.2$ to $t = 4.0$? You must include units in your answer and round to two decimal places.

(b) (3 points) Estimate the **instantaneous rate of change** of the pile's height at $t = 3.6$. Use an estimate of left and right intervals. You must include units in your answer and round to two decimal places.

2. (3 points) Approximate $s'(2)$ using $h = 0.01$ if $s(x) = \sin(\ln(x + 4))$. (Be sure that your calculator is set in radians and round your answer to three decimal places.)

3. (6 points) Use the graph of $p'(x)$ below to answer questions about $p(x)$, $p'(x)$, and $p''(x)$. Circle all points that apply for each question. Choose NA if none of the points apply. CAUTION: **The graph below is that of $p'(x)$ and not $p(x)$.**



(a) At which point(s), if any, is $p(x)$ increasing?

A B C D E NA

(b) At which point(s), if any, is $p'(x)$ increasing?

A B C D E NA

(c) At which point(s), if any, is $p''(x)$ negative?

A B C D E NA

(d) Which point(s), if any, are critical points of $p(x)$?

A B C D E NA

(e) Which point(s), if any, are local minimums of $p(x)$?

A B C D E NA

(f) Which point(s), if any, are local maximums of $p(x)$?

A B C D E NA

4. Let P(t) represent the number of people currently infected with a new type of flu virus in the state of Washington, where $t = 1$ is January 1, 2025.

   (a) (2 points) Data shows that $P(30) = 487$. Explain the meaning of this statement about the new flu using a complete sentence without calculus terms.

   (b) (2 points) Data also shows that $P'(30) = -19$. Explain the meaning of this statement about the new flu using a complete sentence without calculus terms.

   (c) (3 points) Using the values in the previous parts, calculate an estimate for $P(28)$. Explain the meaning of this statement about the new flu using a complete sentence without calculus terms.

5. Find the derivatives below. You are not required to simplify your final answer.

   (a) (3 points) $f'(t)$ for $f(t) = sin(\pi t + 1) - \pi$

   (b) (3 points) $g'(x)$ for $g(x) = \sqrt{x} + x^5 - \frac{2}{x^3}$

   (c) (3 points) $r'(x)$ for $r(x) = e^{cos(3x-1)}$

   (d) (3 points) $m'(t)$ for $m(t) = (3^t)(ln(t))$

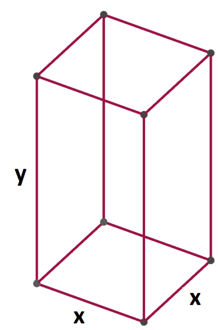
6. For the questions below, use the function $w(t) = 13 - 3t^2 - t^3$ on $-4 \leq t \leq 0$.

   (a) (4 points) Find all critical points of $w(t)$ in its domain.

   (b) (3 points) Classify each of the critical points from part (a) as a local maximum, local minimum, or neither. You must show evidence using derivative tests to receive any credit.

   (c) (4 points) Find any inflection points of $w(t)$. You must use calculus to confirm that the points are actually inflection points. You must show all work and reasoning to receive any credit.

7. A company is designing a storage unit in the shape of a rectangular box with a square top and bottom (see the diagram below). The box must hold a volume of 27,000 cubic meters, and designers want to minimize the box's surface area to lower construction costs.



(a) (2 points) Write a formula for the surface area of the box. Assume the width of the square base is x and the height of the box is y.

(b) (2 points) Write a formula for the total volume of the box. Assume the width of the square base is x and the height of the box is y.

(c) (2 points) Write a formula for the box's surface area in terms of a single variable.

(d) (4 points) **Use calculus** to find the minimum required (i.e. the global minimum) surface area of the box. You must show all calculations and formulas used to receive credit.

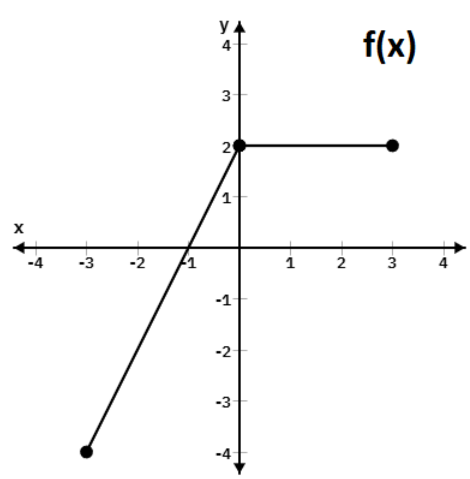
8. Maria runs a chemical reagent business. She calculates her monthly revenue using the function $R(q) = 2100 \cdot \ln(10q + 20)$ and her monthly costs using the function $C(q) = 5000 + 30q$, where q stands for the number of chemical containers produced.

   (a) (2 points) Write a formula for the profit Maria earns every month.

   (b) (3 points) Use calculus to determine the quantity of containers (q) she needs to sell in a month to maximize her profit.

   (c) (1 point) Calculate the profit she expects to make in a month by selling the quantity determined in part (b). Round your answer to the nearest dollar.

9. (5 points) Using the figure below, find $\int_{-3}^{3} f(x)\, dx$. Show all of your work and calculations.



$\int_{-3}^{3} f(x)\, dx =$

10. A polluting chemical is being removed from a large pool at a rate of $r(t)$ (in liters per hour). The data in the table below gives the rate at varying times since the removal process began.

t	0	3	6	9	12
$r(t)$	8.7	7.2	6.0	5.2	4.7

(a) (3 points) Use the table to estimate $\int_0^{12} r(t)\,dt$. Use $n = 4$ and an average of the upper and lower estimates.

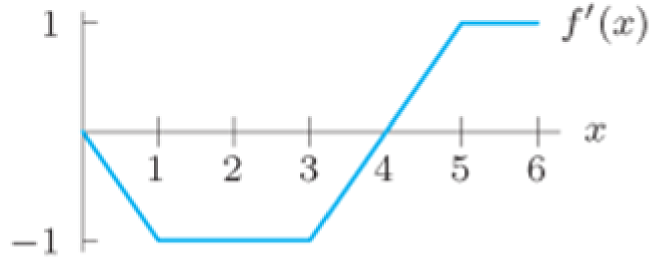
(b) (2 points) Interpret your answer from part (a) in terms of chemical removal. You must include units.

(c) (1 point) If the large pool originally contained 322 liters of the polluting chemical, how much remains after 12 hours?

11. (4 points) Given that $\int_1^7 f(x)\,dx = 4$, $\int_7^{10} f(x)\,dx = -9$, and $\int_1^{10} g(x)\,dx = -8$, evaluate the expression below.

$$\int_1^{10} (2f(x) + 3g(x))\,dx$$

12. (4 points) The graph of $f'(x)$ is shown in the figure below. Given that $f(0) = 50$ find $f(6)$.



13. Find the indefinite integrals below.

(a) (3 points) $\int (\sqrt{x} - \frac{8}{x} - 5x)\,dx$

(b) (3 points) $\int (3sin(x) - 24x^3 - 9)\,dx$

14. (6 points) For $0 \leq x \leq 4$ find the **exact area** between $y = 2x^2 - 18$ and the x-axis using the **Fundamental Theorem of Calculus**. You must show all work and reasoning for credit.

Formula Page

- $ax^2 + bx + c = 0 \Rightarrow x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Area of a rectangle = Length x Width
- Area of a triangle = 1/2 x Base x Height
- Box surface area = Sum of the areas of all sides
- Rectangular box volume = Length x Width x Height
- $\dfrac{d}{dx}\left(cf(x)\right) = cf'(x)$
- $\dfrac{d}{dx}\left(f(x) \pm g(x)\right) = f'(x) \pm g'(x)$
- $\dfrac{d}{dx}\left(f(x) \cdot g(x)\right) = f'(x)g(x) + f(x)g'(x)$
- $\dfrac{d}{dx}\left(\dfrac{f(x)}{g(x)}\right) = \dfrac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$
- $\dfrac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$
- $\dfrac{d}{dx}(c) = 0$, if c is a constant
- $\dfrac{d}{dx}\left(x^n\right) = nx^{n-1}$
- $\dfrac{d}{dx}\left(a^x\right) = a^x \cdot \ln(a)$, if $a > 0$
- $\dfrac{d}{dx}\left(e^x\right) = e^x$
- $\dfrac{d}{dx}\left(e^{kx}\right) = k \cdot e^{kx}$, if k is a constant
- $\dfrac{d}{dx}\left(ln(x)\right) = \dfrac{1}{x}$
- $\dfrac{d}{dx}\left(\sin(x)\right) = \cos(x)$
- $\dfrac{d}{dx}\left(\cos(x)\right) = -\sin(x)$
- $\displaystyle\int k\, dx = kx + C$, if k is a constant
- $\displaystyle\int x^n\, dx = \frac{x^{n+1}}{n+1} + C$, when $n \neq -1$
- $\displaystyle\int a^x\, dx = \frac{a^x}{\ln(a)} + C$, if $a > 0$
- $\displaystyle\int e^x\, dx = e^x + C$
- $\displaystyle\int e^{kx}\, dx = \frac{1}{k} e^x + C$, if k is a constant
- $\displaystyle\int \sin(x)\, dx = -\cos(x) + C$
- $\displaystyle\int \cos(x)\, dx = \sin(x) + C$
- $\displaystyle\int \frac{1}{x}\, dx = \ln(|x|) + C$
- $\displaystyle\int c\, f(x)\, dx = c \int f(x)\, dx$, if c is a constant
- $\displaystyle\int f(x) \pm g(x)\, dx = \int f(x)\, dx \pm \int g(x)\, dx$